PHILOSOPHICAL

TRANSACTIONS.

I. *Observations on the Graduation of Astronomical Instruments; with an Explanation of the Method invented by the late Mr.* Henry Hindley, *of* York, *Clock-maker, to divide Circles into any given Number of Parts. By Mr.* John Smeaton, *F. R. S.; communicated by* Henry Cavendish, *Esq. F. R. S. and S. A.*

Read November 17, 1785.

PERHAPS no part of the science of Mechanics has been cultivated by the ingenious with more assiduity, or more deservedly so, than the art of dividing Circles for the purpose of Astronomy and Navigation. It is said, that TYCHO BRAHE and HEVELIUS laboured this part of their inſtruments with their own hands; and though public rewards have at length brought forth different methods of dividing from our beſt artiſts, which have been communicated to the Public; yet I truſt it will be thought, that if any thing relative to this buſineſs remains yet behind, that may tend to furniſh the ingenious artiſts, who are cultivating this field, with any new or curious idea upon the ſubject, it will be well worth communicating to this learned Society: ſince, if an hint, which is eſſentially different from any thing that (ſo far as I know) the Public is in poſſeſſion of, be once ſtarted, and is purſued and worked upon by ingenious men, it is not poſſible to ſay, to what valuable purpoſes it may be converted.

This, perhaps, will better appear by taking a ſhort review of the labours of others, from the time of TYCHO BRAHE and HEVELIUS (who did not uſe teleſcopic ſights) to the preſent time.

The very learned, ingenious, and inventive Dr. HOOK, in his Animadverſions on the *Machina Cœleſtis* of HEVELIUS, publiſhed in the year 1674, has given us an elaborate deſcription of a quadrant, whoſe diviſions were formed, and afterwards read off, by means of an endleſs ſcrew, working upon the outermoſt border of the limb of a quadrant; which, he ſays, *does not at all depend upon the care and diligence of the inſtrument-maker in dividing, graving, or numbering the diviſions, for the ſame ſcrew makes it from end to end*; yet he has given us no account of any particular care or caution that he uſed, in preventing the ſame ſcrew from making larger or ſmaller paces, in conſequence of unequal reſiſtance, from a different hardneſs of the metal in different parts of the limb; nor any method of correcting or checking the ſame; nor of making a ſcrew, the angle of whoſe threads with the axis ſhall be equal in every part of the circumference; therefore the whole of this buſineſs (in which accurate *mechaniſts* well know conſiſts the whole of the difficulty) he refers to the *ingenious workman*; and, in particular, to the then celebrated Mr. TOMPION, whom, he ſays, he employed to make his inſtrument, and who had thereby *ſeen and experienced the difficulties that do occur therein*: but was any ingenious workman now to purſue the directions of Dr. HOOK, ſo far as his communication thereof extends, we may conclude, that he would make a very inaccurate piece of work, far inferior in performance to what the Doctor ſeems to expect from it *. But yet, I believe, it was the *firſt* attempt to apply the endleſs ſcrew and wheel, or arch, to the purpoſe of forming diviſions for aſtronomical inſtruments; for, the Doctor ſays himſelf, the perfection of this inſtrument is the *way* of making the diviſions; that it *excels all the common ways of diviſion*: and in the table of contents it is intituled, *An Explication of the new Way of dividing.*

This method, however, of Dr. HOOK's was not laid aſide without a very full and ſufficient trial: for Mr. FLAMSTEED, in the *Prolegomena* of the third volume of *Hiſtoria Cœleſtis*, informs us, that *he* contrived the ſextant, wherewith his obſervations were chiefly made, from his entrance into the Royal Obſervatory in the year 1676 to the year 1689. This ſextant was firſt made of wood, and afterwards of iron, with a braſs limb of two inches broad, by Mr. TOMPION, at the expence of Sir JONAS MOORE; the radius thereof was 6 feet 9¼ inches; it was furniſhed with an endleſs ſcrew upon its limb of 17

* This was indeed verified in an attempt upon the ſame plan by the Duc DE CHAULNES, publiſhed in a Memoir of the Royal Academy of Sciences at Paris, for the year 1765.

threads in an inch, and with telescopic sights*. Of this instrument Mr. FLAMSTEED gives the figure at the latter end of his *Prolegomena* before-mentioned, sufficiently large to see the general design; the whole being mounted upon a strong *polar axis* of iron, of three inches diameter.

Though, in the full description of this instrument, Mr. FLAMSTEED mentioned the Limb's being furnished with *diagonal divisions*, distinguishing the arch to 10 seconds †; yet it is pretty clear, that it had not these originally upon it; but that the dependance was wholly upon the screw divisions, when it came out of Mr. TOMPION's hands. This one may reasonably infer from the observations themselves; for the first observation, set down as taken with this instrument, being upon the 29th of October, 1676, it was not till the 11th of September, 1677, that the column which contained the *check* angle by diagonal lines was filled up; and there was also a space of time, antecedent to that last mentioned, wherein no observations are recorded as taken with this instrument, in which time the diagonal divisions might be put on; and this will be put beyond a doubt, as he says expressly, that finding, in the year 1677, that the threads of the screw had worn the border of the limb, he divided the limb into degrees himself, and drew a set of diagonal divisions ‡; and then comparing the two sets of divisions together, he sometimes found them to differ a whole minute; wherefore, for correction thereof, he constructed a new table for conversion of the revolutions and parts of the screw into degrees, minutes, and seconds; and

* "—Qualem nemo, cœlo adhibens;—" Preface to Historia Cœlest. printed in one vol. 1712.

† Prolegomena Histor. Cœlest. vol. III. p. 104.

‡ Ibid. p. 106. "Gradus in limbo destribui ac diagonales duxi."

which

which he applied in the obſervations taken in 1678.—However, notwithſtanding this correction, in looking over the obſervations noted down as deduced each way, I frequently find a difference of half a minute; not unfrequently 40″; but in an obſervation of the moon, of the 9th June, 1687, I find a difference of 55″*, which upon a radius of 6 feet 9 inches amounts to more than $\frac{1}{50}$th part of an inch.

In the year 1689, Mr. FLAMSTEED compleated his *mural arc* at GREENWICH; and, in the *Prolegomena* before-mentioned, he makes an ample acknowledgement of the particular aſſiſtance, care, and induſtry of Mr. ABRAHAM SHARP; whom, in the month of Auguſt, 1688, he brought into the obſervatory, as his *amanuenſis*; and being, as Mr. FLAMSTEED tells us, not only a very ſkilful mathematician, but exceedingly expert in mechanical operations †, he was principally employed in the conſtruction of the mural arc; which in the compaſs of fourteen months he finiſhed, ſo greatly to the ſatisfaction of Mr. FLAMSTEED, that he ſpeaks of him in the higheſt terms of praiſe ‡.

This celebrated Inſtrument, of which he alſo gives the figure at the end of the *Prolegomena*, was of the radius of 6 feet 7½ inches; and, in like manner as the ſextant, was furniſhed both

* Vol. I. of Hiſt. Cœleſt. p. 343.

† "Qui mechanices perquam expertus, pariter ac matheſeos peritus." Prolegomena, vol. III. p. 108.

‡ "SHARPEIUS ſervus meus fideliſſimus, ac omnibus quidem dotibus & facul-
" tatibus erat imbutus, quæ ipſum tam ſubtili & difficili operi obeundo idoneum
" redderent." Prolegom. ibid.

And on finiſhing the inſtrument, he ſays, "Gradus deſcribuntur ſive nume-
" rantur et exſculpantur, artificioſa manuali opera dicti domini SHARP, qui limbum
" partitus eſt, diagonales duxit, totumque organum abſolvit et perfecit: adeo
" ut præſtantiſſimi quivis artifices poſtquam illud conſpexerunt et conſiderarunt,
" ſe exactius id peragere non potuiſſe, agnoverint." Prolegom. p. 111.

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with ſcrew and diagonal diviſions, all performed by the accurate hand of Mr. SHARP. But yet, whoever compares the different parts of the table for converſion of the revolutions and parts of the ſcrew belonging to the mural arc into degrees minutes and ſeconds *, with each other, at the ſame diſtance from the zenith on different ſides; and with their halves, quarters, &c. will find as notable a diſagreement of the ſcrew-work from the hand-diviſions, as had appeared before in the work of Mr. TOMPION: and hence we may conclude, that the method of Dr. HOOK, being executed by two ſuch maſterly hands as TOMPION and SHARP, and found defective, is in reality not to be depended upon in nice matters.

From the account of Mr. FLAMSTEED it appears alſo, that Mr. SHARP obtained the zenith point of the inſtrument, or *line of collimation*, by obſervation of the zenith ſtars, with the face of the inſtrument on the eaſt and on the weſt ſide of the wall †: and that having made the index ſtronger (to prevent flexure) than that of the ſextant, and thereby heavier, he contrived, by means of pullies and balancing weights, to relieve the hand that was to move it from a great part of its gravity †.

I have been the more particular relating to Mr. SHARP, in the buſineſs of conſtructing this mural arc; not only becauſe we may ſuppoſe it the firſt good and valid inſtrument of the kind, but becauſe I look upon Mr. SHARP to have been the firſt perſon that cut accurate and delicate diviſions upon aſtronomical inſtruments; of which, independent of Mr. FLAMSTEED's teſtimony, there ſtill remain conſiderable proofs: for, after leaving Mr. FLAMSTEED, and quitting the department above-mentioned ‡, he

* Hiſt. Cœleſt. vol. II. Appendix. † Prolegom. p. 109.

‡ Mr. SHARP continued in ſtrict correſpondence with Mr. FLAMSTEED ſo long as he lived, as appeared by letters of Mr. FLAMSTEED's found after Mr. SHARP's death; many of which I have ſeen.

he retired into *Yorkſhire*, to the village of *Little Horton*, near *Bradford*, where he ended his days about the year 1743; and where I have ſeen not only a large and very fine collection of mechanical tools (the principal ones being made with his own hands), but alſo a great variety of ſcales and inſtruments made therewith, both in wood and braſs, the diviſions whereof were ſo exquiſite, as would not diſcredit the firſt artiſts of the preſent times: and I believe there is now remaining a quadrant, of four or five feet radius, framed of wood, but the limb covered with a braſs plate; the ſubdiviſions being done by diagonals, the lines of which are as finely cut as thoſe upon the quadrants at Greenwich. The delicacy of Mr. Sharp's hand will indeed permanently appear from the copper-plates in a quarto book, publiſhed in the year 1718, intituled, *Geometry improved by A. Sharp, Philomath.* whereof not only the geometrical lines upon the plates, but the whole of the engraving of letters and figures, were done by himſelf, as I was told by a perſon in the mathematical line, who very frequently attended Mr. Sharp in the latter part of his life. I therefore look upon Mr. Sharp as the firſt perſon that brought the affair of hand diviſion to any degree of perfection.

Some time about the eſtabliſhment of the mural arc at Greenwich, the celebrated Daniſh Aſtronomer Olaus Roemer began his domeſtic Obſervatory, which he finiſhed in the year 1715, as we are informed by his hiſtorian Peter Horrebow, in the third volume of his works, in the tract, intituled, *Baſis Aſtronomiæ*, publiſhed in the year 1741. In this tract is the deſcription of an inſtrument, Tab. III. which not only anſwered the purpoſe of the meridian arc; but, its teleſcope being mounted upon a long axis, became alſo in reality what we now call a *Tranſit Inſtrument*; and which furniſhed, ſo far as I have been able to learn, the firſt idea thereof. One end of the axis of this inſtrument being the center of the meridian arc, and carrying its index, M. ROEMER thereby avoided the errors ariſing from the plane of the mural arc not being accurately a vertical plane; and which Mr. FLAMSTEED endeavoured to check, by obſerving the paſſage of known ſtars nearly in the ſame parallel of declination; that is, paſſing nearly over the ſame part of the plane of the arc; by which he was enabled to correct or check the errors of the arc in right aſcenſion. But it is the peculiar method in which ROEMER *divided* his inſtruments, that occaſions him here to be introduced.

Though it is a very ſimple problem by which geometricians teach how to divide a given right line into any number of parts required; yet it is ſtill a much more ſimple thing to ſet off upon a given right line, from a point given, any number of equal parts required, where the total length is not exactly limited; for this amounts to nothing more than aſſuming a convenient opening of the compaſſes, and beginning at the point given, to ſet off the opening of the compaſſes as many times in ſucceſſion, as there are equal parts required; which proceſs is as applicable to the arch of a circle as it is to a right line. Of this ſimple principle ROEMER endeavoured to avail himſelf.

For this purpoſe M. ROEMER took two ſtiff, but very fine-pointed, pieces of ſteel, and fixed them together, ſo as to avoid, as much as poſſible, every degree of ſpring that would neceſſarily attend long-legged compaſſes, or even thoſe of the ſhorteſt and ſtiffeſt kind when the points are brought near together. The diſtance of the points that he choſe was about the $\frac{1}{10}$ or $\frac{1}{12}$ of an inch. This, upon a radius of $2\frac{1}{2}$ or three feet, would be about 10 minutes. With this opening, beginning

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at the point given, he ſet off equal ſpaces in ſucceſſion to the end of his arch, which was about 75°. Thoſe were diſtinguiſhed upon the limb of the inſtrument by very fine points, which were referred to by a groſſer diviſion, the whole being properly numbered. The ſubdiviſion of thoſe arches of 10 minutes each was performed by a double microſcope, carried upon the arm or radius of the inſtrument, the common focus being furniſhed with parallel threads of ſingle ſilk, whereof eleven being diſpoſed at ten equal intervals, comprehending together one ten-minute diviſion, the diſtance of the neareſt threads became a very viſible ſpace, anſwerable to one minute each, and therefore capable of a much further ſubdiviſion by eſtimation.

The diviſions of this inſtrument were therefore, properly ſpeaking, not degrees and minutes; but yet, if exactly equal, would ſerve the purpoſe as well, when their true value was found, which was done by compariſon with larger inſtruments.

Now, if it be conſidered, that in going ſtep by ſtep of ten minutes each, through a ſpace of 75 degrees, there will be a ſucceſſion of 450 diviſions, dependant upon each other; if it be alſo conſidered, that the leaſt degree of extuberance in the ſurface of the metal, where each new point is ſet down, or the leaſt hard particle (wherewith all the baſe metals ſeem to abound) will cauſe a deviation in the firſt impreſſion of a taper point, and thereby produce an inequality in the diviſion; it is evident, that though this inequality may be very ſmall, and even imperceptible between neighbouring diviſions, yet among diſtant ones, it may and will ariſe to ſomething conſiderable; which, in the menſuration of angles, will have the ſame ill tendency as in near ones. Now, as M. ROEMER has given us no

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means of checking the distant divisions, in respect of each other, it is very probable that no one has followed his steps, in cases where great accuracy was required, in a considerable number of divisions. For in reality this method is likely to fall far short of Dr. HOOK's; as Dr. HOOK's divisions being cut in a similar successive manner, by the rotation of the sharp *edge* of the threads of a screw against the exterior edge of the limb of the instrument, a very slight degree of pressure will bring a fine screw of thirty threads in an inch (which he prescribes) to touch against an arch whose radius is four or five feet in more than one, two, three, or four threads at once; so that the threads supporting one another, a small extuberance, or even a small hard particle in the metal, will be cut through or removed by the grinding or rather sawing motion of the screw; and which, in regard to its contact, being in reality an edge, will be much more effectual (that is, more firm) in its retention than a mere simple point: and a repetition of the operation, from the support of the threads to each other, will tend to mend the first traces; whereas, in ROEMER's way, a repetition will make them worse; for, whatever drove forward or backward the point on first entering, will, from the sloping of the point, be confirmed and increased in driving it deeper.

When Dr. HALLEY was chosen Astronomer Royal (Mr. FLAMSTEED's instruments being taken away by his executors), Mr. GRAHAM undertook to make a new mural quadrant, about the year 1725; who, uniting all that appeared valuable in the different methods of his predecessors, executed it with a degree of contrivance, accuracy, and precision, before unknown: and the division thereof he performed with his own hand. The model of this quadrant, for strength, easy management, and convenience, has been ever since pursued as the most perfect.

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What I apprehend to be peculiar in it, was the application of the arch of 96°; not only as a check upon the arc of degrees and minutes, but as ſuperior thereto, being derived from the more ſimple principle of *continual biſection*.

To make room for this, he has entirely rejected the ſubdiviſion by diagonals, and has adopted the method of the *Vernier*; but the ſubdiviſion of the vernier diviſions he, as I apprehend for the firſt time, meaſured by the turns of the detached adjuſting ſcrew, making it in fact a micrometer, by which the diſtance of the *ſet* of the inſtrument was to be meaſured from the perfect coincidence of one of the actual diviſions of the limb with the next ſtroke of the vernier; by which means the obſervation could not only be read off with all the preciſion that the diviſion of the inſtrument was capable of, but the two ſets of diviſions could be checked and compared with each other. Another thing that I apprehend to be peculiar in this inſtrument, was the more certain method of transferring and cutting the diviſions, from the original divided points, by means of the *beam-compaſs*, than could poſſibly be done from a *fiducial edge*, as had doubtleſs been conſtantly the practice in cutting diagonals; for, placing the ſteady point of the beam-compaſs in the tangent line to that part of the arc where each diviſion was to be cut, the opening of the compaſs being nearly the length of the tangent, the other point would cut the diviſion in the direction of the radius nearly; and though in reality an arch of a circle, yet the ſmall part of it in uſe would be ſo nearly a right line, as perfectly to anſwer the ſame end; all which advantages put together, it is probable, induced Mr. Graham to reject the diagonals.

Soon after the completion of this quadrant, Mr. Graham undertook to execute a *zenith ſector* for the Rev. Dr. Bradley, which was fixed up at *Wanſtead*, in *Eſſex*, in the year 1727. The very ſimple conſtruction that he adopted for this inſtrument (the plumb line itſelf being the index) did not admit of the uſe of a vernier: he therefore contented himſelf with dividing the arch of the limb of this inſtrument by primary points, as cloſe as he thought neceſſary, that is, by diviſions of five minutes each, and meaſuring the diſtance from the *ſet* of the inſtrument to the next point of diviſion by a *micrometer* ſcrew, in the conſtruction of which ſcrew he uſed uncommon care and delicacy. I have mentioned this inſtrument to introduce this obſervation; that I think it highly probable, had Mr. GRAHAM conſtructed the great quadrant *after* the zenith ſector had been fully tried, he would have rejected not only the diagonals but the verniers alſo, as containing a ſource of error within themſelves which may be avoided by a well-made ſcrew*.

It ſeems alſo, that Mr. GRAHAM, at the time he conſtructed both theſe inſtruments, was not aware how much error could ariſe from the unequal expanſions of different metals by heat or cold: for in both the radius, or frame of the inſtrument, was iron, while the limbs were of braſs. They, however, remain in the Royal Obſervatory, perfect models, in all other reſpects, of every thing that is likely to be attained in their reſpective deſtinations, and monuments of the ſuperlative abilities of that great mechanician Mr. GRAHAM †.

* This has been found conſentaneous to the experience of my friend Mr. AUBERT, who, on my ſuggeſtion, has long ſince laid aſide the uſe of his vernier, meaſuring always by the micrometer ſcrews the diſtance between the ſet of the inſtrument, and the coincidence of the firſt ſtroke of the vernier with the next primary diviſion of the limb.

† I have been informed, that Dr. MASKELYNE has cauſed this objection to the ſector to be rectified, ſince its removal to the Royal Obſervatory, by ſubſtituting an iron limb inſtead of that of braſs, the points being made upon ſtuds of gold.

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Mr. GRAHAM lived till the year 1751; and during his time there were few inſtruments of conſequence conſtructed without his advice and opinion. They were for many years done by Mr. SISSON, to whom doubtleſs Mr. GRAHAM would fully communicate his method of diviſion; and from this ſchool aroſe that very eminent and accurate artiſt Mr. BIRD, whoſe delicate hand, joined with great care and aſſiduity, enabled him ſtill further to promote this branch of diviſion; and which being carried by him to a great pitch of perfection, the Commiſſioners of Longitude did themſelves the credit, by an handſome reward, to induce him to publiſh to the world his particular method of dividing aſtronomical inſtruments; which being drawn up by himſelf, in the year 1767, this matter is fully ſet forth to the public: I ſhall therefore only take this opportunity of obſerving, that there ſeems to be one article in which Mr. BIRD's method may be ſtill improved.

I muſt here obſerve, that I apprehend no quadrant, that has ever undergone a ſevere examination, has been found to form a perfect arch of 90°; nor is it at all neceſſary it ſhould: the perfect equality of the diviſions throughout the whole is the firſt and primary conſideration; as the proportion of error, when aſcertained by proper obſervations, can be as eaſily and readily applied, when the whole error of the rectangle is fifteen ſeconds, as when it is but five.

In this view, from the radius taken, I would compute the chord of ſixteen degrees only. If I had an excellent plain ſcale, I would uſe it; becauſe I ſhould expect the deviation from the right angle to be leſs than if taken from a ſcale of more moderate accuracy; but if not, the equality of the diviſions would not be affected, though taken from any common diagonal ſcale. This chord, ſo prepared, I would lay off five times in ſucceſſion, from

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the primary point of ⊙ given, which would compleat eighty degrees; I would then bifect each of thofe arches of 16°, as prefcribed by Mr. BIRD, and laying off one of them beyond the 80th, would give the 88th degree: proceeding then by bifection, till I came to an arch of two degrees, laying that off from the 88th degree, would give the point of ninety degrees. Proceeding ftill by bifection, till I had reduced the degrees into quarters = fifteen minutes each, I would there ftop; as from experience I know, that when divifions are over clofe, the accuracy of them, even by bifection, cannot be fo well attained as where they are moderately large. If a fpace of $\frac{4}{10}$ of an inch, which is a quarter of a degree, upon an eight-feet radius, is thought too large an interval to draw the index over by the micrometer fcrew, this may be fhortened by placing another line at the diftance of one-third of a divifion on each fide of the index line, in which cafe the fcrew will never have to move the index plate more than one-third of a divifion, or five minutes; and the perfect equality of thofe fide lines from the index line may be obtained, and adjufted to five minutes precifely, by putting each of the fide lines upon a little plate, capable of adjuftment to its true diftance from the middle one, by an adjufting fcrew.

The above hint is not confined to the chord of fixteen degrees, which prohibits the fubdivifions going lower than fifteen minutes: for if it be required to have divifions equivalent to five minutes upon the limb itfelf; then I would compute the chord of 21° 20′ *only*; and laying it off four times from the primary point, the laft would mark out the divifion 85° 20′, pointed out by Mr. BIRD; *fupplying the remainder* to a quadrant, from the bifected divifions as they arife, and not by the application of other *computed* chords.

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In my Introduction to M. ROEMER's Method of Division, I have shewn, that divisions laid off in succession, by the same opening of the compasses, either in a right line, or in the arch of a circle, being in its idea geometrically true, and in itself the most simple of all processes, it has the fairest chance of being mechanically and practically exact, when cleared of the disturbing causes. The objection therefore to his method is, the great number of repetitions, which depending upon each other in succession (requiring no less than 540 to a quadrant, when subdivided to ten minutes each), the smallest error in each, repeated 540 times, without any thing to check it by the way, may arise to a very sensible and large amount: but in the method I have hinted, this objection will not lie; for, in the first case, the assumed opening is laid off but five times; and in the latter case but four times; nor does this *repetition* arise out of the nature of the thing; for, if you like it better, you may, in the former case, at once compute the chord of 64°; and in the latter that of 85° 20′, and then proceed wholly by bisection; supplying what is wanted to make up the quadrant, from the bisected divisions, as they arise. Mr. BIRD prescribes this method himself, for the division of HADLEY's sextants and octants.

He, I suppose, was the first, who conceived the idea of laying off chords of arches, whose subdivisions should be come at by continual bisection; but why he mixed therewith divisions that were derived from a different origin (as prescribed in his method of dividing) I do not easily conceive. He says, that after he had proceeded by the bisections, from the arc of 85° 20′, the several points of 30°, 60°, 75°, and 90°, (all of which were laid down from the principle of the chord of 60° being equal to radius), *fell in without sensible inequality*; and so indeed they might; but yet it does not follow that they were equally true in their places

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as if they had been (like the reſt) laid down from the biſection from 85° 20′, and therefore being the firſt made, whatever error was in them, would be communicated to all connected with them, or taking their departure from them. Every heterogeneous mixture ſhould be avoided where equal diviſions are required. It is not the ſame thing (as every good artiſt will ſee) whether you *twice* take a meaſure from a ſcale as *nearly* the ſame as you can, and lay them off ſeparately; or lay off *two* openings of the compaſſes, in ſucceſſion, *unaltered*; for though the ſame opening, carefully taken off from the ſame ſcale a ſecond time, will doubtleſs fall into the points made by the firſt, without ſenſible error; yet as the ſloping ſides of the conical cavities made by the firſt point will conduct the points themſelves to the center, there may be an error which, though inſenſible to the ſight, would have been avoided by the *more ſimple proceſs* of laying off the opening twice, without ever altering the compaſſes.

The 96 arc was, I have no doubt, invented by Mr. GRAHAM, from having perceived, in common with all preceding artiſts, how very much more eaſy a given line was to biſect, than to triſect, or quinqueſect; and therefore the 96 arc which proceeded by biſections only (or by laying off the ſame identical openings, which, as already ſhewn, is ſtill more ſimple and unexceptionable) was wholly intended by him by way of checking the diviſion of the arc of 90, which required triſections and quinqueſections. But experience ſoon ſhewed the ſuperior advantage of it ſo ſtrongly, that the uſe of the 90 arc is now wholly ſet aſide, where accuracy is required; whereas the ingenuity of Mr. BIRD having ſhewn a way to produce the 90 arc by biſection, when this is really purſued quite through the piece, by rejecting all diviſions derived from any other origin, the 90 arc will have nothing in it to prevent its being

equally

equally unexceptionable with the 96 arc; and conſequently if, inſtead of the 96 arc, another arc of 90 was laid down (which being upon a different radius, its diviſions will ſtand totally unconnected with the former), then theſe two arcs would in *reality* be a check upon each other; for being of equal validity, the mean might be taken: and if (in lieu of vernier diviſions) ſtrokes at the diſtance of any odd number, as 7, 9, 11, or 13, are marked upon, and carried along with, the index plate; theſe will produce a check upon neighbouring diviſions; and the angle may then be deduced from the medium of no leſs than four readings.

The laſt works that have been made known to the public in the line of graduation (ſo far as has come to my knowledge) are thoſe of the very ingenious Mr. Ramsden, which were publiſhed, by order of the Board of Longitude, in the year 1777.

From his own information, I learn, that in the year 1760 he turned his thoughts towards making an engine for dividing mathematical inſtruments; and this he did in conſequence of a reward offered by the Board of Longitude to Mr. Bird, for publiſhing his method of graduating quadrants; for as, ſeveral years previous to that period, he had taken great pains to accompliſh himſelf in the art of hand-dividing, in which line Mr. Bird had acquired his eminence, he conceived by this publication of Mr. Bird's he ſhould be reduced to the ſame ſtandard of performance with the reſt of the trade. He, therefore, partly to ſave time, and that kind of wearineſs to an ingenious mind that ever muſt attend the endleſs repetition of the ſame thing from morning to night; partly ſtill to preſerve the pre-eminence he had then gained; and partly to procure diſpatch in the great increaſe of demand for Hadley's ſextants and octants, in conſequence of the ſucceſsful application of the moon's motion to the

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purpose of ascertaining the longitude at sea (which instruments for this purpose required a degree of accuracy and certainty in the division, by no means necessary thereto when applied to the simple purpose of observing latitudes); I say, for these considerations, Mr. RAMSDEN determined to set about something in the instrumental way, that should be sufficient effectually to answer these purposes.

Accordingly, considering the nature of the endless screw, he set himself to work upon an engine whose divided wheel or plate was of thirty inches diameter; and though the performance of this first essay was inferior to his expectations and wishes, yet with it he was able to divide theodolites with a degree of precision far superior to any thing of the kind that had been exhibited to the public.

This engine I myself saw in the spring of the year 1768; and it appeared to me not only a very laudable attempt towards instrumental divisions, but a very good model for the construction of an engine of the most accurate kind for that purpose. And he furthermore, at the same time, shewed me the model or pattern for casting a wheel of a much larger size, which he proposed to make upon the same plan, and with considerable improvements. This being effected some time in or about the year 1774, its accuracy was proved by making a sextant, afterwards subjected to the examination of Mr. BIRD; who in consequence approved the method, not only as fully sufficient for the division of HADLEY's sextants and octants for any purpose whatsoever, but in fact for dividing any instrument whose radius did not exceed that of the dividing wheel, which was forty-five inches in diameter: whereupon the Board of Longitude, ever ready to encourage all endeavours that tend to the certainty and perfection of any thing subservient to the purpose of finding the longitude at sea, very properly and usefully resolved to confer

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an handſome reward on Mr. RAMSDEN, for delivering a full explanation of his method of making the ſaid engine; which, in conſequence, was publiſhed by order of the Board of Longitude in the year already mentioned, viz. 1777: the deſigns whereof are ſo full and explicit, that whoever could not underſtand that deſcription, ſo as to enable him to make it, would be unfit to undertake it on other accounts.

From what I have ſaid upon the works of the different artiſts that I have mentioned, it would ſeem that the art of graduation was brought to that degree of perfection, that nothing material can now be added thereto: and I ſhould have been apt to have thought ſo myſelf, if I had not happened, in the courſe of my life, to have had a communication made to me (under the ſeal of ſecrecy) which ſeems to promiſe yet further light and aſſiſtance in perfecting that important art; and every impediment to the diſcovery thereof being now removed, I ſhall in the remainder of this eſſay give the cleareſt deſcription thereof that I am able, with ſuch elucidations and improvements as ſeem to be naturally pointed out by the method itſelf.

In the autumn of the year 1741, I was firſt introduced to the acquaintance of that then eminent artiſt, Mr. HENRY HINDLEY, of *York*, clockmaker;—he immediately entered with me into the greateſt freedom of communication, which founded a friendſhip that laſted to his death, which did not happen till the year 1771, at the age of 70. On the firſt interview, he ſhewed me not only his general ſet of tools, but his *engine*, at that time furniſhed with a dividing plate, with a great variety of numbers for cutting the teeth of clock wheels, &c. and alſo, for more nice and curious purpoſes, furniſhed with a wheel of about thirteen inches diameter, very ſtout and ſtrong, and cut into 360 teeth; to which was applied an endleſs ſcrew, adapted thereto. The threads of this ſcrew were not formed upon a cylindric ſurface, but upon a ſolid whoſe ſides were terminated by arches of circles. The whole length contained fifteen threads; and as every thread (on the ſide next the wheel) pointed towards the center thereof, the whole fifteen were in contact together; and had been ſo ground with the wheel, that, to my great aſtoniſhment, I found the ſcrew would turn round with the utmoſt freedom, interlocked with the teeth of the wheel, and would draw the wheel round without any ſhake or ſticking, or the leaſt ſenſation of inequality.

How long this engine might have been made before this firſt interview, I cannot now exactly aſcertain: I believe not more than about a couple of years; but this I well remember, that he then ſhewed me an inſtrument intended for aſtronomical purpoſes, which muſt have been produced from the engine, and which of itſelf muſt have taken ſome time in making *.

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* This inſtrument was of the equatorial kind; the wheel parallel to the equator, the quadrant of latitude, and ſemi-circle of declination, being all furniſhed with ſcrews containing fifteen threads each, framed and moved in the ſame manner as that of the engine; the whole of which inſtrument was already framed, and the teleſcope tube in its place, which was intended to be of the inverting refracting kind, and to be furniſhed with a micrometer. This, however, was not compleated till ſome years after; but, in the year 1748, I received it in London for ſale. It ſtaid with me two years, in which time I ſhewed it to all my mechanical and philoſophical friends, amongſt whom was Mr. SHORT, who afterwards publiſhed in the Philoſophical Tranſactions, vol. XLVI. N° 493. p. 241. an Account of a portable Obſervatory, but without claiming any particular merit from the contrivance. However, the model of it differs from HINDLEY's equatorial only in the following articles. He added an azimuth circle and compaſs at the bottom. He omitted the endleſs ſcrews, placing verniers in their ſtead; and at the top, a reflecting teleſcope inſtead of a refractor. This inſtrument of HINDLEY's being afterwards returned to him unſold, I pointed out the principal deficiencies that I found therein; viz. that, in putting the inſtrument into different poſitions,

I in reality thought myſelf much indebted to Mr. HINDLEY for this communication; but though he ſhewed me his engine, and told me, that the ſcrew was cut by the rotation of the point of a tool, carried round upon a ſtrong arm, at the diſtance of the radius of the wheel from the center of motion, which arm was carried forward by the wheel itſelf, and the wheel was put forward by an endleſs ſcrew, formed upon a cylinder to a proper ſize of thread, cut by his chock lathe; though he ſhewed me alſo this chock lathe, and the method employed to make the threads of the ſcrew *equiangular* with the axis, that is, to free the ſcrew from what workmen term *drunkenneſs*; and alſo ſhewed me how, by the ſingle ſcrew of his lathe, he could cut, by means of wheel-work, ſcrews of every neceſſary degree of fineneſs * (and, by taking out a wheel, could cut a left-handed ſcrew of the very ſame degree of fineneſs); by which means he was enabled not only to adapt his plain ſcrew to the ſize of the teeth of his wheel, but alſo to prevent any drunkenneſs that otherwiſe the curved ſcrew would be ſubject to in conſequence of being produced from the plain

tions, the ſpringing of the materials was ſuch as in ſome poſitions to amount to conſiderable errors. This remained with him in the ſame ſtate till the year of the firſt *Tranſit of Venus*, viz. 1761; when it was ſold to —— CONSTABLE, Eſq. of *Burton Conſtable*, in *Holderneſs*. Mr. HINDLEY, to remedy the evil above-mentioned, applied balances to the different movements. He ſoon afterwards completed one, *de novo*, upon this improved plan, for his Grace the late Duke of NORFOLK. A method of balancing in much the ſame way, without the knowledge that it had been done before, has been fully explained, and laid before the Society, by our ingenious and worthy brother Mr. NAIRNE. Phil. Tranſ. vol. LXI. p. 108.

* A machine for cutting the endleſs ſcrew of Mr. RAMSDEN's engine, upon principles exactly ſimilar, is fully and accurately ſet forth in his Deſcription of his dividing Engine above-mentioned.

one; furthermore, that the ſcrew and wheel, being ground together as an optic glaſs to its tool, produced that degree of ſmoothneſs in its motion that I obſerved; and, laſtly, that the wheel was cut from the dividing plate: yet, how the dividing plate was produced, he for particular reaſons reſerved to himſelf.

Nor can he be blamed for the reſervation of this one ſecret; as he had, even at the time of my early acquaintance with him, a kind of foreſight that from the ſuperior merit of HADLEY's quadrant, a demand for that, and other inſtruments for the purpoſe of navigation, was likely to increaſe; and that he might live to ſee a public reward offered for a method of dividing them with greater accuracy and diſpatch than had at that time appeared. Indeed, he had himſelf an idea, from the ſatisfactory ſucceſs that had attended his operations in dividing, that a ſcrew and wheel, produced from his engines of one foot diameter, would have as much truth as the eight-feet quadrant at Greenwich: and though he doubtleſs greatly over-rated the accuracy of theſe miniature performances, yet it does not follow, as his methods were not confined to ſo narrow a compaſs, but that, his ſcale of operation being proportionably enlarged, a degree of accuracy in the graduation of aſtronomical inſtruments may be attained in proportion.

I muſt here beg leave to obſerve, that there appears to me to be a natural limitation to the accuracy of inſtruments, conſiſting of conſiderable portions of a circle, ſuch as quadrants, &c.*. I do not find that the fineſt ſtroke upon the limb of a quadrant, made by BIRD's own hand, if removed from its

* The zenith ſector conſiſts but of *few* degrees, with little variation of its poſition in uſing it.

coincidence with its index, can be replaced with any degree of certainty nearer than the 4000dth part of an inch, though aided by a magnifying glass *.

A 4000dth part of an inch being then determined to be the *minimum visibile* by the strokes of an instrument, this will be less than one second of a degree upon a radius of four feet; and therefore, if the whole set of divisions upon the limb could be preserved true to this aliquot part of an inch, the eight-feet quadrants of Greenwich might be expected to be true to half a second. How far they are from this, I do not exactly know; but I have reason to think they vary from it some seconds: nay, I believe it is generally allowed, that our largest quadrants, even when executed by the accurate hand of Mr. BIRD, do not exceed those of a less size, by the same hand, in proportion to their increase of radius: nor can it well be expected that they should; since, as the weight necessarily increases in a triplicate ratio of the radius, the great weight of the Greenwich quadrants in moving and fixing them (as they could not be divided in their place) may easily derange the framing; or even the *internal elasticity* of the materials may give way, by a change of position, to so minute a quantity as a 4000dth part of an inch. It therefore appears to me, that since the divisions of a quadrant of four-feet radius are more than sufficient, and even those of three feet admit of all the distinctness that in other respects is wanted, a three-feet quadrant, in point of

* It will be to little purpose to attempt it with a greater power. Double microscopes can doubtless be formed to magnify objects, far less than a 4000dth part of an inch, to distinct surfaces; but then the advantage of such degrees of magnifying power is chiefly upon the organized bodies of nature. Let a dot, or the finest point that can be made by human art, be so viewed, and it will appear not round, but a very ragged irregular figure.

size,

ſize, is capable of all attainable exactneſs; and would be as much to be depended on as any of thoſe now in being of eight feet. By adopting quadrants of this ſmaller ſize, we ſhall of courſe get rid of $\frac{18}{19}$ths of the preſent weight; and conſequently of much cumber, unhandineſs, and derangement, that muſt ariſe from that weight, as well as the fear of totally diſcompoſing them, if ever moved out of their place.

It now comes to be time to open a principle upon which there is a proſpect of effecting ſuch an improvement. I have ſhewn that a 4000th part of an inch is the *ultimatum* that we are to expect from *ſight*, though aided by glaſſes, when obſerving the diviſions of an inſtrument. But in the XLVIIIth volume of the Philoſophical Tranſactions for the year 1754, I have ſhewn the mechaniſm of a new *pyrometer*, and experiments made therewith; whereby it appears, that, upon the principle of *contact*, a 24,000th part of an inch is a very definite quantity. I remembered very well that I did not then go to the extent of what I might have aſſerted, being willing to keep within the bounds of *credibility*: but on occaſion of the preſent ſubject, I have re-examined this inſtrument, and find myſelf very well authoriſed to ſay, that a 60,000th part of an inch, with ſuch an inſtrument, is a more definite and certain quantity than a 4000th part of an inch is to the *ſight*, conditioned as above ſpecified. The certainty of contact is, therefore, fifteen times greater than that of viſion, when applied to the diviſions of an inſtrument; and if this principle of certainty in contact did not take place even much beyond the limit I have now aſſigned, we never ſhould have ſeen thoſe exquiſite mirrors for reflecting teleſcopes, that have already been produced.

Theſe reflections apply immediately to my preſent ſubject, as HINDLEY's method of diviſion proceeds *wholly by contact*, and that of the firmeſt kind; there being ſcarcely need of magnifying glaſſes in any part of the operation.

In the year 1748 I came to ſettle in London; and the firſt employment I met with was that of making philoſophical inſtruments and apparatus. In this ſituation, my friend HINDLEY, from a principle the reverſe of jealouſy, fully communicated to me, by letter, his method of diviſion; and though I was enjoined ſecrecy reſpecting others (for the reaſons already mentioned), yet the communication was expreſsly made with an intention that I might apply it to my own purpoſes.

The following are extracts from two letters, which contain the whole of what related to this ſubject; and ſince I have many things to obſerve thereon, ſo that the paraphraſe would be much greater than the text, I think it beſt not to interrupt the deſcription with any commentary, as perhaps his own mode of expreſſion will more briefly and happily convey the general idea of the work than any I can uſe inſtead of it.

MY DEAR FRIEND, York, 14 Nov. 1748.

AS to what you was mentioning about my brother's knowing how I divided my engine plate, I will deſcribe it as well as I can myſelf; but you will want a good many things to go through with it.

The manner is this: firſt chuſe the largeſt number you want, and then chuſe a long plate of thin braſs; mine was about one inch in breadth, and eight feet in length, which I bent like an hoop for an hogſhead, and ſoldered the ends together; and turned it of equal thickneſs, upon a block of ſmooth-grained wood, upon my great lathe in the air (that is, upon the end of the mandrel): one ſide of the hoop muſt be rather wider than

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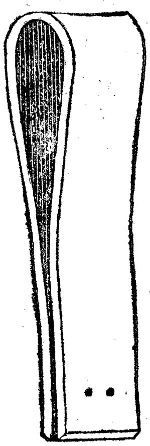
the other, that it may fit the better to the block, which will be a ſhort piece of a cone of a large diameter: when the hoop was turned, I took it off, cut, and opened it ſtraight again.



The next ſtep was to have a piece of ſteel bended into the form as *per margin* *; which had two ſmall holes bored in it, of equal bigneſs, one to receive a ſmall pin, and the other a drill of equal ſize. I ground the holes after they were hardened, to make them round and ſmooth. The chaps formed by this ſteel plate were as near together as juſt to let the long plate through. Being open at one end, the chaps ſo formed would ſpring a little, and would preſs the long plate cloſe, by ſetting in the *viſe*. Then I put the long plate to a right angle to the length of the ſteel chaps, and bored one hole through the long plate, into which I put the ſmall pin; then bored through the other hole; and by moving the ſteel chaps a hole forward, and putting in the pin in the laſt hole, I proceeded till I had divided the whole length of the plate.

The next thing was to make this into a circle again. After the plate was cut off at the end of the intended number, I then proceeded to join the ends, which I did thus: I bored two narrow ſhort braſs plates † as I did the long one, and put one on the inſide, and the other on the outſide of the hoop, whoſe ends were brought together; and put two or three turned ſcrew pins, with flat heads and nuts to them, into each end, which held them together till I rivetted two little plates, one on each ſide of the narrow plate, on the outſide of the hoop. Then I took out the ſcrews, and turned my block down, till the hoop

* The figure is conſiderably leſs than the real tool ſhould be.

† Theſe I ſhall hereafter diſtinguiſh by the name of *ſaddle-plates.*

would fit cloſe on; and by that means my right line was made into an equal divided circle of what number I pleaſed.

The engine plate was fixed on the face of the block, with a ſteel hole fixed before it, to bore through; and I had a point that would fall into the holes of the divided hoop; ſo by cutting ſhorter, and turning the block leſs, I got all the numbers on my plate.

I need not tell you, that you get as many prime numbers as you pleaſe; nor that the diſtance of the holes in the ſteel chaps muſt be proportioned to the length of the hoop.

You may aſk my brother what he knows about my method of dividing; but need not tell him what I have ſaid about it; for I think neither *he* nor *John Smith* know ſo much as I have *told you*, though I believe they got ſome knowledge of it in general terms *.——I deſire you to keep the method of dividing to *yourſelf*, and conclude with my beſt wiſhes,

and am, dear Sir, yours, &c.

HEN. HINDLEY.

Though the above letter was in itſelf very clear and explicit, as to the general traces of the method, yet ſome doubts occurring to me, a further explanation became neceſſary. A copy of my letter not being preſerved, the purport of it may be inferred from the anſwer, which was as follows:

* The perſons here referred to were both bred with him. His brother, Mr. ROGER HINDLEY, who has many years followed the ingenious profeſſion of a watch-cap-maker in London, was ſo much younger as to be an apprentice to him. Mr. JOHN SMITH, now dead, had ſome years paſt the honour to work in the inſtrument way, under the direction of the late Dr. DEMAINBRAY, for his preſent MAJESTY.

DEAR FRIEND, York, 13 March, 1748-9.

I THINK in your laſt you ſeem to be apprehenſive of ſome difficulties in drilling the hoop for dividing: Firſt, that the center of the hole in the hoop might not be preciſely in the center of the hole of the ſteel chaps, it was drilled in; but if I deſcribed fully to you the method I uſed, I can ſee no danger of error there: for my chaps were very thick, and the two correſponding holes were a little conical, and ground with a ſteel pin; firſt one pair, and then the other, alternately, till the pin would go the ſame depth into each. Then for drilling the hoop, I took any common drill that would paſs through, and bore the hole. After that I took a five-ſided broach, which opened the hole in the braſs betwixt the ſteel chaps, but would not touch the ſteel; ſo conſequently the center of the holes in the braſs muſt be concentric with the holes in the chaps: and for alterations by air, heat, cold, &c. I was not above two or three hours in drilling a row of holes, as far as I remember.

2dly, For drilling, in a right line, I had a thin braſs plate, faſtened between the ſteel chaps, for the edge of the hoop to bear againſt, whilſt I thruſt it forward from hole to hole. What you propoſe of an iron frame with a lead outſide, will be better than my wooden block; but conſidering the little time that paſt, betwixt transferring the diviſions of the hoop to the diviſions of my dividing plate, I did not ſuffer much that way. It was when I drilled the holes in my dividing plate that I uſed a frame for drilling, which had one part of it that had a ſteel hole, that in lying upon the plane of the dividing plate was fixed faſt in its place for the point of the drill to paſs through: then, at the length of the drill, there was another piece of ſteel, with a hole in it, to receive the other end of the drill to keep it at right-angles to the plane of the plate. This piece was a ſpring, which bended at the end, where it was faſtened to the frame of the lathe, at about eighteen inches from the end of the drill; ſo it puſhed the drill through with any given force the drill would bear: and though that end of the drill moved in the arch of a circle, it was a very ſmall part of it, being no more than equal to the thickneſs of the dividing plate.

My good wiſhes conclude me yours,

HEN. HINDLEY.

Whoever attentively conſiders the communication contained in the above letters will ſee, that more happy expedients could not have been deviſed to procure a ſet of diviſions, where there ſhould be the moſt exact equality among *neighbours*; and which, for the purpoſes of clock-making, is the principal thing to be wiſhed for. But herein, as in M. ROEMER's method, there were no means of checking the diſtant diviſions, which run on to 360: now ſuch a check, when the expanſion of metals is conſidered, and particularly the difference of expanſion between braſs and ſteel, ſeems abſolutely neceſſary for the purpoſe of diviſions upon inſtruments, where the accurate menſuration of large angles is required, as much as the equality of neighbouring diviſions *.

With this view the invention of this ingenious perſon ſuggeſted to him the thought of making his curved ſcrew to lay

* The ingenious Mr. STANCLIFFE (ſome years a workman of HINDLEY's) has ſuggeſted, that the difference of expanſion between the ſteel chaps and the braſs hoop may be avoided by making the chaps of braſs alſo, with hard ſteel holes ſet ſeparately therein, ſomewhat ſimilar to the jewelled holes of watches.

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hold of fifteen teeth or degrees together: this, in effect, becomes a pair of compaſſes, 24 removes of which complete the whole circle, and produce 24 checks in the circumference: and whoever conſiders the very exquiſite degree of truth that reſults from the grinding of ſurfaces in contact, as already noticed, muſt expect a very great degree of rectification of whatever errors might ſubſiſt in the wheel after its firſt cutting.

What degree of truth it might in reality be capable of upon its firſt production and adjuſtment, is not now to be aſcertained, he never having uſed it for the graduation of any capital inſtrument. Thoſe that he made with a view to an accurate meaſure of angles, he always made with a ſcrew and wheel, or parts of circles cut by his engine into teeth, and ground together as before-mentioned; but I have reaſon to think, that its performance, if put to a ſtrict teſt, was never capable of that accuracy that he himſelf ſuppoſed it to have.

The method itſelf, however, from its ſimplicity, and eaſe of execution, ſeems to me to be a foundation for every thing that can be expected in truth of graduation; and in conſequence for reducing inſtruments to the leaſt ſize that is capable of bringing out all that can be expected from the largeſt; when it ſhall, like manual diviſion, have received thoſe advantages that the joint labours of the moſt ingenious men can beſtow upon it. That I may not appear to be without grounds for my expectations, I will beg leave to propoſe, what near forty years occaſional contemplation has ſuggeſted to me on the ſubject; and as I can deſcribe the proceſs I would purſue, where different from HINDLEY's, in fewer words than I could make out a regular criticiſm upon his letters, I will immediately proceed to the deſcription of it.

Proposed Improvements *of* Hindley's *Method.*

I would recommend the number of parts into which the circle is to be reduced to be 1440; that is 4 times 360; which divisions will therefore be quarters of a degree; the distances of the holes in the chaps will therefore, to a three-feet radius, be $\frac{157}{1000}$ of an inch nearly; that is, between the one-sixth and one-seventh of an inch distance center and center.

Having provided myself with a stout mandrel, or arbor, for a *chock Lathe*, properly framed, that would turn a circle of six feet diameter, I would prepare a chock, or platform, for the end of it, of that diameter, or a little more, composed of clean-grained mahogany plank, all cut out of the same log; which, when finished, to be about $1\frac{1}{2}$ inch thick, and formed in sectors of circles, suppose 16 to make the circle; the middle line of each sector lying in the direction of the grain of the wood, this will consequently every where point outward: the method of framing this kind of work is well known.

The way of getting a slip of brass to answer the circumference of this platform is suggested in Mr. Bird's Account of constructing Mural Quadrants. Let a parallelogram of brass of about three feet long, and of a competent substance (suppose half an inch) to make it when finished about one-twentieth of an inch in thickness, be cast of the finest brass; and this to be rolled down till it becomes of sufficient length for the hoop, and about one-fifth part more. I would then cut off, from the whole length, somewhat better than one-sixth part, the whole being sufficiently reduced to a thickness by the rollers. Perhaps no way will be more ready and convenient to

get

get ſuch a long ſtrip of braſs reduced to an equal breadth, than the method preſcribed by HINDLEY; viz. by turning it upon the chock prepared; but I would not make it wider on one ſide than the other, like the hoop of a caſk, as he deſcribes, but exactly to fit the chock, when truly cylindric; for the internal elaſticity of the braſs, in ſo great a length, will be very ſufficient for fitting it on tight enough, without any tapering. This I will now ſuppoſe done; and a pair of ſteel *chaps*, as deſcribed by HINDLEY, to be alſo prepared, and ready for grinding; which, by ſuch a careful admeaſurement as can eaſily be made, will give the length of the hoop ſufficiently near, on its firſt preparation.

Method of forming a Pair of Straps *as a* check *to the Diviſions.*

The part firſt cut off muſt be again cut into two equal parts in length; which, for diſtinction ſake, I will call the *ſtraps*; and which are to ſerve as checks to every 60th and every 120th diviſion of the circle.

A ſteel plate, of about half an inch in breadth, the ſame thickneſs as the ſtraps, and in length equal to the breadth of the hoop plate, muſt be ſoldered with ſilver ſolder to one end of each of the ſtraps, by which means their length will be increaſed half an inch by the ſteel. An hole muſt then be made through each ſteel plate, of the ſame ſize as thoſe through the chaps, and anſwerable to the middle of the ſtraps; but ſo near the border of the ſteel, that when the chaps are put on, and adapted to the ſteel hole, the next hole will fall through the braſs. The ſteel plates muſt then be hardened; and a pin being put through the two holes and the two plates, theſe muſt

be

be wrought to a right line in contiguity to each other; by this means the ſtraight edge of each of the ſtraps will be reduced to the ſame diſtance from the ſteel hole: the hard ſteel edges may be rectified by the grindſtone, if neceſſary.

This being done, not only the holes in the chaps, but the holes in the two ſteel plates, applied to each other, like the two ſides of the chaps, muſt be reſpectively ground together; not with a taper pin, as preſcribed by HINDLEY; but ſo as not only to be cylindrical, but that the ſame cylindrical pin ſhall equally fit them all, and leave them ſmooth and poliſhed; which is a proceſs no ways difficult to a curious artiſt, and of which therefore a minute deſcription is unneceſſary.

The chaps being then put upon one of the ſtraps, with its ſtraight edge uppermoſt, and a pin put through the holes on the left-hand, and through the ſteel hole in the ſtrap under operation, the chaps muſt be ſet upright, ſo that the line joining the centers of the holes ſhall be parallel to the upper edge of the ſtrap; the braſs plate, mentioned by HINDLEY, between the chaps, as a guide for directing them always to that upright poſition, may be then adjuſted and fixed to the inſide of the chap next the operator *.

The performance of the enſuing part of this work ſhould be at a ſeaſon when the temper of the air is not very variable; rather above the mean temper (ſuppoſe at 60°) than below it;

* It would be well, previous to the drilling of the ſteel chaps, that another hole was drilled in the chaps, that ſhould be ſomewhat above the upper edge of the ſtraps, and in the middle betwixt ſide and ſide, to receive a *ſteady pin* therein, antecedent to drilling the main holes; for then a tempered ſteel pin, a little taper, will, by driving it in as far as neceſſary, conſtantly anſwer this purpoſe from firſt to laſt, ſo as to regulate the holes in grinding, to be truly oppoſite: proper holes ſhould alſo be drilled for fixing the braſs guide plate to one of the chaps.

 but

but above all things the artiſt ſhould be himſelf cool; that is, not in a ſtate of ſenſible perſpiration; and there ſhould be a free circulation of air in the room. Things being thus conditioned in reſpect to temperature, he may begin to drill the holes in one of the ſtraps; the pin being firſt put through the chaps and through the ſteel hole of the ſtrap; and the next hole, being drilled through the braſs with a common drill, that and every hole as it goes is to be finiſhed with a taper broach, as preſcribed by HINDLEY; and he may then prove or finiſh every hole by the application of a thorough broach, made ſo full as to require a degree of preſſure to force it through; and this broach being a little tempered, and the holes quite hard, there will be no fear of injuring the ſteel holes *.

Calling the hole in the ſteel plates o, and obſerving the time of beginning, you may proceed to drill 60 holes as preſcribed by HINDLEY; and noting how long you have been about it, you may lay the work aſide a length of time, equal to the time you took in drilling; that any addition of warmth it may have acquired in handling or working may be again loſt in a great degree †. After this pauſe you may begin again, and go on to finiſh 60 holes more; that is, to the length of

* The ſteel holes in the chaps need not to be above one-twentieth of an inch in diameter; and though it may be proper to make the ſteel plate, of which they are formed, one tenth of an inch thick, in order to give the ſpring formed between them a convenient degree of ſtiffneſs, yet they may be reduced (by chamfering the outſides) to half that thickneſs.

† As there is not much occaſion for the artiſt to touch his work, the effects of that may alſo be very much avoided by wearing thick gloves; and the friction being but ſlight, and the work almoſt continually in the viſe, the variation of temperature in the metals concerned cannot be ſenſible or conſiderable.

120 holes from the beginning; you then proceed in the ſame manner with the other ſtrap.

Method of drilling *the Hoop.*

You are now prepared to commence the work upon the long or *hoop-plate*; and you proceed therewith, in forming the firſt hole with the chaps, as before directed by HINDLEY, and this firſt hole you call o. You then place the ſtraps one on each ſide the hoop, with their gaged edges upward, and put the pin through the holes denominated 60 upon the ſtraps, and through the firſt hole already made, and denominated o upon the hoop; then, bringing the gaged edges of the ſteel plates to be even with the upper or working ſide of the hoop, you pinch them together in the viſe, and drill and broach the hole through the ſteel plates, which will make the hole, number 60, upon the hoop. This done, you put the pin through the left-hand hole of the chaps, and the hole marked o upon the hoop-plate firſt made, and proceed to drill with the chaps to 59 holes incluſive, which will fill up the whole ſpace from o to the 60th diviſion before obtained.

You now again have recourſe to the ſtraps, and placing them one on each ſide the hoop-plate, you put the pin through the 120th hole of the ſtraps, and through the hole marked o upon the hoop-plate; and regulating the ſteel plates to the hoop-plate as before, you drill and form a hole with the ſteel plates, which will correſpond with the 120th hole upon the hoop-plate; and afterwards filling up the 59 holes wanting, by means of the chaps, you then have all completed to the 120th diviſion, which is one-twelfth of the whole circle.

You then proceed, in like manner, with another set of 120 holes; that is, placing the 60th hole of the straps to the 120th hole of the hoop-plate, and from it producing the 180th hole; you, in like manner as before, fill up this 60 with the chaps; and afterwards placing the 120th hole on the straps in the 120th hole on the hoop-plate, you will obtain the 240th hole; so that filling up this last set of 60 divisions, you have obtained 241 holes, including 240 spaces or divisions of the hoop; and repeating this process ten times more, you will, in like manner, obtain 1441 holes, comprehending 1440 spaces*. And this process being carried on in temperate weather, the manner of working produces twelve similar operations, wherein the materials and tools concerned will not only be subject to very little change of temperature, but that change, whatever it is, will be nearly similar in each set of 120 holes: we may therefore infer, that the greatest inequality, or indeed any that can be sensible, must be at every 60 divisions, that is, between the 59th and 60th, and between the 119th and 120th, both which will be equally repeated 12 times, in the whole length which is to compose the *circumference of a Circle*, and which will thus be checked thereby 12 times in the circumference, and 12 times more at the intermediate distances; that is, with 12 master checks, and 12 subordinate ones, in the whole round.

It is proper here to observe, that in M. ROEMER's method even sixty divisions could scarcely be trusted in an affair of great accuracy, on account of the objections already made, arising from the points having such slight hold in the surface of the brass; but here the parts are held so exceedingly firm, and the

* It will be proper, for reasons hereafter to be mentioned, to continue the divisions to 20 holes more, making in the whole 1461 holes.

operation carried on with ſo much power, that any ſmall inequality in the hardneſs of the braſs, or irregularity of ſurface, cannot be ſuppoſed to affect the place of the center of the hole; nor will any ſmall inequality that may be ſuſpected from the wear of the ſteel holes ſenſibly affect the *center* of the hole, to which every thing is ultimately referred.

Method of joining *the Hoop.*

A more happy thought than that of HINDLEY's, for joining the two ends of the hoop, could ſcarcely have been wiſhed for, in regard to preſerving the ſame equality of the ſpace between the holes contiguous to the joint, as in the other parts: for though, geometrically ſpeaking, the two *ſaddle* plates, in which the little cylindrical bolts are fixed, for bringing the terminating holes of the hoop plate to their due diſtance, being one applied within the hoop, and the other without, will belong to circles of different *radii*; yet this difference being exceedingly ſmall in ſuch thin metal, and ſo great a radius, and one being as much too big for the hoop as the other is too little, when the bolts are put in, and the hoop in that part ſet nearly to a circle by a mould; the mean between them aſſumed by the hoop, from the elaſtic compreſſibility of the materials, will be the truth.

It muſt, however, be remarked, that in the uſe of the ſtraps, the joining of the hoop ſhould not be made at any part betwixt an 119th and an 120th diviſion, as ſome inequality muſt be ſuppoſed there, unleſs the ſaddle plates were adapted thereto. The method the moſt eaſily practiſed, will be to continue the diviſion upon the hoop, about twenty more than the completion of the number intended to form the circle, and to cut off all the overplus ones at the beginning.

The faddle plates I would recommend to contain ten holes each; fo that if the divifions are carried on to twenty more than what will be contained in the circle, there will be a piece containing twenty to cut off; and this again being cut in the middle will afford ten holes to make each faddle plate; fo that there will be a place for a bolt on each fide the joint, and then putting a bolt through every other hole, there will be three bolts at an end.

The pieces deftined for the faddle plates, thus obtained, being broader than can be admitted when put to this ufe, I would advife to divide the breadth of the plate into three equal parts; and with a cutting hook (which perhaps will be attended with the leaft violence in the feparation) to feparate the two outfide pieces from the middle piece: by this means the two faddle plates (though double) will occupy one third only of the breadth of the hoop in the middle; and two of the pieces cut off being applied, one on each fide of the faddle plate on the outfide, will anfwer in like manner for the *rivet* plates.

The laft operation to compleat the joining of the hoop is the putting on the rivet plates: to compleat this, I would advife a piece of brafs, of three or four inches in length, to be filed fo as to anfwer to the infide of the hoop, when reduced to a true circular form; and being three-eighths, or one-half an inch in thicknefs, to file the oppofite fide fomewhat nearly concentric thereto; apply the middle of its convex arch to the infide of the hoop at the joint, and then bringing on the middle of one of the rivet plates to the joint of the hoop, confine the three together by a couple of narrow-chapped hand vifes, leaving a fpace between them capable of receiving a couple of pins as rivets on each fide the joint; the holes for the rivets are then to be drilled through all, and a little fmoothed with a broach at

their

their entry, into which ſmooth taper pins are to be driven; not with violence, but moderately, that no ſenſible ſtretching of the ſolid parts may take place thereby; then cutting off and ſmoothing the heads, ſhift the viſes ſo as to receive another couple of holes, and a third couple in the ſame end of the hoop; and proceed progreſſively in the ſame manner, from the middle to the other end of the rivet plate; then gently ſeparate the internal braſs mould with a thin knife, or ſuch like inſtrument; and cutting off, and very lightly rivetting the inner ends, proceed to fix the other rivet plate, in the ſame manner, on the other ſide: by this means the hoop will be firmly joined in the very poſition given it by the ſaddle plates and mould. Theſe plates may then be removed, the inſide of the hoop cleared and ſmoothed, if neceſſary; and the outſide will have the middle part clear where the diviſions lie, and that without ſenſible loſs or gain in the juncture.

Method of transferring *the Diviſions of the Hoop to a dividing Plate.*

The hoop being thus refitted for the chock, that ſhould be turned down to leave a ſhoulder on one ſide, that the hoop, now reduced to an equal breadth, may be forced againſt it; and the diviſions, being equally diſtant from one of its edges, will be all found in a circle, as if turned upon it. It ſhould be very carefully fitted to the chock, that it may go on with a ſufficient degree of tightneſs, and without the neceſſity of much forcing; and it will be no inconvenience now, if it goes on upon a very ſlight degree of taper of the chock, as the internal ſpring of the materials will eaſily accommodate it to this ſhape without any injury to its general truth: a ſlight degree of a groove ſhould

be

be turned in the place where the diviſions will come, that any conical pin, that is to ſerve as an index, let drop into the diviſions or holes, may not, by reaching through this thin plate, abut upon the wood, rather than upon the ſides of the holes: and thus this hoop is made into a wheel of 1440 equal diviſions, moveable round upon its own axis, whereon it was formed.

Againſt the time that this is compleated, there muſt be prepared a flat circular plate or wheel of braſs, the rim of which ſhould be of about 3½ inches breadth, and about two-tenths of an inch in thickneſs when finiſhed, to make a dividing plate; the external diameter of this is to be ſuch, that when laid flat upon the ſurface of the mahogany platform, its extreme edge will exceed the diameter of the hoop by about half an inch all round. There muſt alſo be prepared braſs arms (ſuppoſe eight in number) of an equal ſubſtance with the outer rim, and all connected with a circular plate in the middle; and, the whole of this work being framed beforehand, is to be let on flat upon the mahogany platform; whoſe face is ſuppoſed to be turned truly flat, and ſufficiently affixed with ſcrews: in this ſituation, the outward edge is to be turned, and the outward face of the rim turned flat. The *center plate*, which may be about twelve inches diameter, is alſo to be turned as flat as poſſible, and a center hole, of about half an inch diameter, to be very carefully turned therein.

A piece of clean, ſtraight-grained, well ſeaſoned mahogany, of about two feet long, three inches thick, and five or ſix inches broad, is then to be well affixed to ſome part of the general frame of the lathe, which muſt now have its poſition altered, ſo that the platform will become horizontal; and therefore the frame ſhould

be

be originally made with this view *. The piece of mahogany is to be affixed ſo that one of its larger faces ſhall be in a parallel plane to the face of the platform, and ſo low as to clear the under ſide of the platform in its rotation; and ſo far diſtant from the center, that an index may be fixed upon this upper face of the piece of wood, ſo as conveniently to drop into the holes of the hoop; while the common *cutter frame* of a clock-maker's engine ſhall be firmly attached upon the ſame face of the wood, and ſo fixed as to cut the edge of the dividing plate into teeth, anſwerable to the ſeveral diviſions of the hoop. The teeth need only to be cut with a common cutter, making a parallel notch: and here it will be proper to obſerve, that not only both the index and cutter are to be founded on the ſame piece or baſe of wood; but that the nearer they are together, the more free they will be from the effects of all variations of expanſions by variations of temperature †.

The equaliſing *the Teeth of the dividing Plate by* grinding.

The object of transferring the diviſions of the hoop to the teeth of the dividing plate, is ſtill farther to equaliſe the teeth by grinding; eſpecially thoſe that, falling within the compaſs of

* After changing the poſition of the lathe, the collar of its mandrel ſhould be removed, and the neck made to move within three planes, ſo as to preſerve an exact center, in the manner of an *equal altitude* inſtrument.

† It is proper to obſerve, that as it may be impracticable to get the rim of the dividing plate caſt of the proper ſize, in one entire piece, it will be very practicable, if caſt of a leſs ſize (ſuppoſe half), but of a ſufficient thickneſs, to roll it down; and by having the outward edge originally thicker than the inner, in the proportion of the *radii*, it may be ſo managed by the rollers as to be of an equal thickneſs when brought to its proper ſize. But the arms and center plate ſhould be of the ſame metal, rolled in the ſame degree.

 each

each ſet of 120 diviſions, may be ſuppoſed, if any, to be mended thereby; but as it may be incommodious to conſtruct a curved ſcrew, of ſuch a length and ſize, in HINDLEY's method, as would be ſufficient for the purpoſe, I would propoſe to uſe two ſcrews of braſs, cut from a cylinder in the way ſet forth by Mr. RAMSDEN, each of which, with a very little grinding upon this large circumference, would lay hold of ten or twelve teeth together. I would place the two ſcrews, that is, their middles, to be ninety diviſions aſunder; of conſequence, when one of the ſcrews is between the 59th and the 60th, or between the 119th and 120th diviſion of each ſet, the other will be in the middle of the ſpace divided by the chaps only *.

The threads of theſe ſcrews I would adviſe to be cut a little taper, ſo that as they grind in, they may fill the notches of the teeth; which alſo, by this means, will acquire a little tapering towards their extremities; and by cutting the notches parallel, as I have mentioned, the true ground part will always be certain of being at the extremity.

When the ſcrews have been uſed in grinding till they are found to have the effect of a perfectly equal and eaſy rotation all round, and all the teeth reduced to a ſenſible taper, and regular bearing, I would then totally remove the ſcrews from the ſquare block of wood, upon whoſe upper face I ſuppoſe them to have been mounted; in like manner as I ſuppoſe the index and cutting frame to have been removed, to make room for the mounting of

* The beſt way of giving an equal motion to thoſe two ſcrews, ſeems to be by a detached axis, carrying two common flat wheels; one acting upon a like flat wheel, upon the axis of one ſcrew, and the other, in the ſame manner, upon the other; and applying the pulley for communicating the power to the middle of the detached axis between the two wheels, the ſpring or twiſt will be equal both ways; ſo that in turning the contrary way round, they will ſtill be in equal advance.

the ſcrews. I now conſider the teeth of the dividing plate, ſo formed, as having all the equality that the preſent known ſtate of human art has pointed out; and the whole convertible upon the axis or mandrel upon which it has been originally formed, and the central hole of the plate concentric therewith: I therefore conſider the ground faces of the teeth of the plate as the actual diviſions. It now remains to ſhew how they are to be transferred, to form the diviſions of an inſtrument.

Preparation *of the dividing Plate for* graduating *Inſtruments.*

If a ſmall cylinder of hard ſteel is duly poliſhed, and made of a ſize ſo as juſt to chock in betwixt the extremities of the teeth, then the center of that cylinder will be a fixed point, in reſpect to the circumference of the wheel: if another cylinder is applied in like manner, at the diſtance of a number of diviſions (ſuppoſe it a prime number, ſo as to croſs all former diviſions, viz. 17 or 19), then the middle of the line joining the centers of the two cylinders will remain in the direction of the *ſame radius*, though one of them ſhould force in a minute quantity further than the other; and if a point is aſſumed in the direction of a tangent to a circle at this middle-point, then though both the cylinders ſhould drop in a minute quantity further at one time than another, yet the middle-point would remain at the ſame diſtance from the point in the tangent; provided that point was removed to a competent diſtance, that is, to five or ſix inches. On this principle I would conſtruct an index, the two cylinders being fixed in a frame, convertible about the middle-point, and to be centered in the end of the lever, repreſenting the tangent; then this lever being again convertible about the point in the tangent line, the middle-point would always have a fixed diſtance

 from

from the point in the tangent, and there hold it ſteadily faſt; the tangent point being placed upon the fixed block before-mentioned.

Uſe *of the dividing Plate in the* Graduation *of Inſtruments.*

Our dividing plate is now ready for the reception of an inſtrument; ſuppoſe it a quadrant, whoſe radius, however, muſt not exceed the radius of the dividing plate: It is to be laid upon the face of the dividing plate, and a weight, or weights, equivalent to that of the quadrant, is placed on the oppoſite ſide, to balance it. It muſt alſo be ſuppoſed, that the quadrant is made with a view to be divided by this engine; and conſequently, that the central cylinder is ſo well adapted, and nicely fitted to the center hole of the quadrant, that the center cylinder can be removed, in order for the limb to be divided, and again replaced, without ſenſibly altering its center. This being the caſe, let a piece of metal be turned, to apply to the quadrant, perfectly like its center cylinder at the upper end, and turned nicely to fit the central hole in the dividing plate, at the lower end; then, the quadrant being fixed with proper faſtening ſcrews, I would cut the diviſions with a beam compaſs; and, if a fixed point is aſſumed, *viz.* the center of the tangent point for the index; then the beam compaſs being always opened to the computed length of the tangent of the circle of diviſions, it will be ſufficiently near for cutting the diviſions, ſquare to the circular arches between which they are placed.

It will alſo be proper (to prevent unequal expanſions) that the beam of the compaſs ſhould be formed of a piece of clean-grained *white fir*; and that the length between the points be incloſed in a tube of tin or braſs; without touching the beam, except

except at the terminations, which will in a great meaſure protect it from both alteration of moiſture, and of heat from the body of the artiſt, during the operation.

It will be likewiſe proper to have a lever, or ſome equivalent contrivance, to bring the dividing plate forward; that after lifting the little cylinders out of the diviſions, and reſting them upon the tops of the teeth, they may be brought gently forward with an equal drag, and ultimately ſnap in between the teeth, by the ſtrength of the ſpring commanding the index; by this means the drag of the friction of the whole will be conſtantly the ſame way.

Concluſion.

Now, if, as it has been ſhewn, a quadrant of any radius may be read off to the 4000dth part of an inch, then this quantity upon a radius of three feet will not be ſo much as 1$\frac{1}{3}$ ſecond; and as the whole of the proceſs is carried on by contact, in which a greater error than that of a 60,000dth part of an inch cannot be admitted in any ſingle operation, I ſhould aſſuredly expect a three-feet quadrant, ſo divided, to be true in its diviſions, and read off to at moſt two ſeconds.

But, after all, in an inſtrument like this, I ſhould expect the greateſt ſource of error to be in the want of perfect coincidence of the center of the diviſions with the actual center upon which the index revolves; and therefore, that if, inſtead of a quadrant of three-feet radius, a complete circle of five feet diameter was divided, and its diviſions read off from the two oppoſite points (taking the mean), then the errors of the center will be wholly avoided. For this reaſon, I am very clearly of opinion, that the ſagacious propoſition of Mr. RAMS-

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DEN, to uſe circles inſtead of quadrants, or other portions of circles, will bid much the faireſt for perfection in actual practice; and that his ingenious method of making them both ſtiff and light, by the uſe of hollow conical tubes by way of ſpokes, in the manner of a common wheel, will enable him to mount them of five feet diameter, upon hollow axes, in the nature of a *tranſit*. By this means we ſhall have all the good properties of both the quadrant and tranſit united in one inſtrument; and obſervations both of right aſcenſion and declination, through the very ſame teleſcope, as long ſince attempted by M. ROEMER; and to a degree of perfection and certainty, in point of declination, hitherto unattainable by the largeſt inſtruments that have yet been made.

N. B. In matters of very nice determination, ſmall circumſtances often come to be of conſequence; and it is in this view that I mention what follows. It was a practice of HINDLEY's of many years ſtanding, and ſince followed by myſelf and others, wherever he made any uſe of the *vernier*, to lay the vernier plate in the ſame plane, or cylindrical ſurface continued, whereon the principal diviſions are cut. It is of equal utility, though the vernier be rejected, to lay the index ſtroke in the plane of the diviſions. In this way the diviſions being by convenience upon the external border of the limb *, *two* ſets of diviſions are thereby rendered incommodious; but thoſe

* It has been objected, that laying the diviſions upon the extreme edge of the limb of the inſtrument ſubjects it to injury: but, to obviate this, in an HADLEY's quadrant made for me, by my direction, by the late Mr. MORGAN, in the year 1756, wherein the vernier is laid even with the diviſions, thoſe are protected by a projection of the ſolid part of the limb, beyond the diviſions; a *Rabbet* being ſunk in the edge of the limb, to clear the vernier.

that

that wiſh two ſets, as a check, will in a great meaſure aid themſelves, by reading from two different parts of the ſame ſet of diviſions; which is very eaſily provided for, by putting an additional ſtroke upon the index plate, at the diſtance of 9, 11, or any prime number of diviſions to 19, 23, or more; and reading off from that ſtroke alſo; as before recommended for great quadrants, where the vernier is propoſed to be rejected *: ſo that they will thereby be mutually checked by diviſions that had no correſpondence in their original formation.

* I would not have it thought, from my propoſal of rejecting the vernier, that I have any quarrel with it; I think it a very ſimple and ingenious contrivance, where it is properly applicable; that is, where the ſtrokes of the vernier, or their eſtimated halves, are ſufficient for all the preciſion required or expected from the inſtrument, as in HADLEY's quadrants, theodolites, &c.: but where ſtill more minute diviſions are required than can eaſily be had by eſtimation from the vernier; to do this by a ſcrew, as a *ſupplement* to the vernier, appears to me in the light of bringing a more accurate tool to ſupply the deficiencies of one leſs accurate; when the former might, with more propriety, ſupply the place of the latter altogether.